Problem Situation! (for students)

The Problem

Off Course!

You are the navigation officer on the LCROSS mission control team. You have just received new tracking data that says the spacecraft is off course and will miss its impact point by 10 kilometers! That is to say, the spacecraft will fall 10 kilometers short of the target crater.

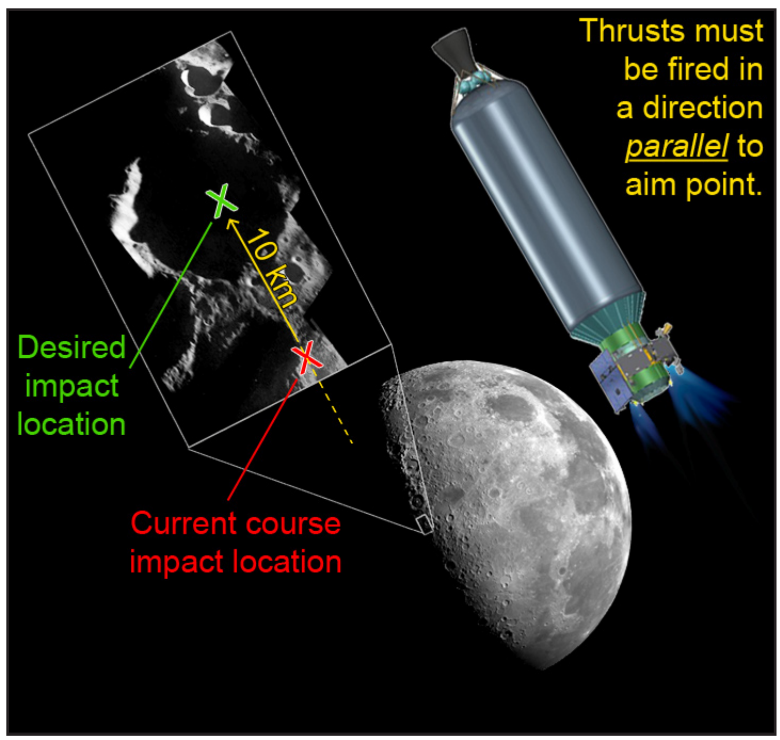
The spacecraft is currently 10 hours from impact on the Moon, and there is a final opportunity to perform a correction maneuver 2 hours from now. If the spacecraft weighs 7,275 pounds, then:

How long must you fire the onboard thrusters to get back on target and save the mission?

Key Concept

As the great English scientist Isaac Newton first stated, moving objects have a property that causes them to continue moving unless acted on by a force. The same property causes an object at rest to remain at rest if no force acts upon it. Newton called this property **inertia**.

On Earth, one of the forces acting on a moving object to slow it down is **friction**. Airplanes experience friction with the air; sailing ships encounter friction from the water; cars are slowed by friction with the road. Yet in the vacuum of space there is no friction. It's as if everything were moving on a perfectly smooth, 3-dimensional ice rink. So, if you fire a spacecraft's thruster, even for just a brief moment, it will cause an increase or decrease in the spacecraft's velocity that will remain in effect indefinitely until the spacecraft is acted upon by another force. Therefore, one way to counteract a burst from a thruster in one direction is to fire a burst in the *opposite* direction.



Important Information

LCROSS has 2 onboard thrusters for trajectory correction maneuvers, each of which produces 5 pounds of thrust. Both thrusters always fire together. To change LCROSS's aim point on the Moon, you must fire its thrusters in a direction *parallel* to the direction you wish to shift the aim point. (See illustration above.)

Problem Solution (for teachers)

Approaching the Problem

The *Problem Situation* on page 31 is for you to present to your students. The *Problem Solution* on pages 32–39 is for you, the teacher, to review to help you understand the science and math used to solve this problem.

The physics concepts associated with this problem are advanced and are more suitable for upper grade levels; however, the mathematics are centered mostly on unit conversion and multiplying and dividing fractions, which is fitting for lower grade levels. You, the teacher, can decide how much or how little the students need to know in order to solve this problem. For example, teachers of upper level students may wish to explore the physics and units of measurement in depth and to work through the solution step by step, whereas teachers of lower level students may choose to give students the numerical values to plug into the equations, making this simply a math problem.

Don't be intimidated by the solution to this problem. It is based on two equations: force equals mass times acceleration (F = m a) and velocity equals acceleration times time (v = a t). We will begin by reviewing each of the physics concepts associated with these two equations. Next, we will rewrite these equations, as needed, and solve for each variable one by one. Then, as the final step, we will derive the amount of *time* (t) that the thrusters need to be fired by dividing velocity (v) by acceleration (a).

In this guide, the solution is worked using both the International System of units (also known as the metric system or SI) and the Customary System of units (also known as English or Standard units). For each equation, you will see the solution worked for both systems side by side. You should choose the one system that is most appropriate for your students; it is not necessary for your students solve the problem for both systems of measurement.

Situation Summary

Ten hours remain before impact, and the spacecraft is currently on a course that will miss the target crater by 10 kilometers. Two hours from now, that is to say at 8 hours from impact, the rocket thrusters will need to be fired so that the spacecraft's course is changed by 10km over a period of 8 hours. The total combined *weight* of the Centaur upper stage and the LCROSS shepherding spacecraft is 7,275 pounds. The spacecraft has two rocket thrusters, and each rocket delivers 5 pounds of thrust for a combined total thrust of 10 pounds of force. This can be summarized as follows:

Required correction:	10 km
Time for correction:	8 hr
Velocity change (delta-V):	10 km / 8 hr
Total spacecraft weight:	7,275 lb
Total rocket thrust:	10 lbs of force

Main Idea

In spaceflight, it is common to make mid-flight course corrections by firing onboard thrusters to change a spacecraft's velocity. The amount of time that the thrusters need to be fired to properly correct the course can be determined using the simple equation:

time = velocity ÷ acceleration

Forces and Motion

There are many factors to consider when solving this problem, including force, mass, weight, acceleration, velocity, and time.

A **force** (F) is anything, such as a push or a pull, that changes an object's velocity or causes an object to move. Force has both a magnitude and a direction. Newton's second law can be stated as "force equals mass times acceleration," meaning the net force on an object is equal to the mass of the object multiplied by its acceleration.

$$F = m\ a$$

Remember, in the frictionless environment of outer space, objects will continue to move along a new path and at a new velocity when acted upon by a force until they are acted upon by another force, in this case the impact.

Mass (m) is a measure of the quantity of matter an object contains. Mass is not a force and is not the same thing as weight. Whether an object is on the surface of the Earth, on the surface of the Moon, or free falling through space, its mass will remain the same (unless, of course, the object itself is changed by adding to or taking away from its mass). In the International System of units (SI), mass is measured in *kilograms* (kg), and in the Customary System of units, mass is measured in *slugs*.

Weight is the force of gravity acting on an object. Since weight (w) is a force and the acceleration of the object being weighed is due to gravity (g), we can rewrite Newton's "F = m a" equation in a special form:

$$w = m\ g$$

Weight is a force and should not be confused with mass. Since the weight of an object is determined by both the object's mass *and* the force of gravity pulling on the object, the object's weight will vary whether it is on the surface of the Earth, on the surface of the Moon, or free falling through space because the force of gravity is different for each of these locations. The standard units of weight are the *newton* (N) for SI and the *pound* (lb) for Customary.

Fun Fact!

Because an object's *mass* does not change, the amount of force needed to move the object does not change either. For example, the same amount of force is required to bowl a bowling ball on the Moon as is required to bowl a bowling ball on Earth.

The *weight* of the bowling ball is less on the Moon than it is on Earth, but the *mass* of the bowling ball is the same in both locations. Therefore the same amount of effort (or force) is needed to move the bowling ball in both locations.

Kilograms vs. Newtons	**Slugs vs. Pounds**
The kilogram (kg) is a unit of mass, and the newton (N) is a unit of weight (force). One newton of force will accelerate a mass of 1 kilogram at a rate of 1 meter per second per second.	*The slug is a unit of mass, and the pound (lb) is a unit of weight (force). One pound of force will accelerate a mass of 1 slug at a rate of 1 foot per second per second.*
$1 \text{ N} = (1 \text{ kg}) \left(\frac{1 \text{ m}}{\text{s}^2}\right)$	$1 \text{ lb} = (1 \text{ slug}) \left(\frac{1 \text{ ft}}{\text{s}^2}\right)$
We can also say that, on Earth, the weight of 1 kilogram is 9.8 newtons. (See table below.)	*We can also say that, on Earth, the weight of 1 slug is 32.2 pounds. (See table below.)*

Velocity is defined as the rate of change of position. It is a vector and can change in two ways: a change in magnitude and/or a change in direction. Velocity is defined by both speed (magnitude) and direction and is measured in meters per second (m/s) in SI units and in feet per second (ft/s) in Customary units. Velocity (v) equals acceleration (a) times time (t) as shown by the following formula:

$$v = a\ t$$

Acceleration is the change in velocity over time. Acceleration is measured in meters per second per second (m/s^2) in SI units and in feet per second per second (ft/s^2) in Customary units.

Fun Fact!

Ignoring air resistance, any and all objects regardless of their weights, when dropped from the same height, will hit the ground at the same time.

You can find images and video of this experiment being conducted on the Moon at:

http://nssdc.gsfc.nasa.gov/planetary/lunar/apollo_15_feather_drop.html

On average, the strength of **gravity** at the Earth's surface (g) is 9.80665 meters per second per second or 32.174049 feet per second per second. These values can be rounded to 9.8 m/s^2 and 32.2 ft/s^2 respectively.

SI	Customary
$g \approx 9.8\ m/s^2$	$g \approx 32.2\ ft/s^2$

This means that, ignoring air resistance, an object falling freely near the Earth's surface increases its velocity by 9.8 m/s (32.2 ft/s) for each second of its descent. Thus, an object starting from rest will attain a velocity of 9.8 m/s (32.2 ft/s) after one second, 19.6 m/s (64.4 ft/s) after two seconds, and so on, adding 9.8 m/s (32.2 ft/s) for each succeeding second.

The **g-force** of an object is its acceleration relative to free fall. **Free fall** is motion with no acceleration other than that provided by gravity. An acceleration of 1 g is generally considered as equal to standard gravity, which on Earth is generally defined as 9.8 m/s^2 (32.2 ft/s^2).

Solving the Problem

This problem can be solved using International units (SI) or Customary units. The important thing to remember is to make sure that the correct units are used for the force and velocity equations, and to convert to appropriate units of measurement when necessary. The two formulas we will use to solve this problem are:

Force equals mass times acceleration	$F = m\ a$
Velocity equals acceleration times time	$v = a\ t$

Let's begin with the force equation. In the equation $F = m\ a$, "force" (F) must be in newtons (N) for SI or in pounds (lb) for Customary and "mass" (m) must be in kilograms (kg) for SI or in slugs for Customary.

In this problem situation, we know that the two onboard rockets will create a combined total of 10 pounds of thrust. This means that the ***force*** of the rocket egines is equal to 10 pounds, or ~ 44.5 newtons.

SI	Customary
Pounds can be converted to newtons using the unit ratio: $\frac{1\text{ N}}{0.2248089\text{ lb}}$ $F = 10\ \cancel{\text{lb}} \cdot \frac{1\text{ N}}{0.2248089\ \cancel{\text{lb}}}$ $F = 44.4822\text{ N}$ $\mathbf{F \approx 44.5\text{ N}}$	$\mathbf{F = 10\text{ lb}}$

We also know that the combined total ***weight*** of the LCROSS and Centaur upper stage is 7,275 pounds.

SI	Customary
Pounds can be converted to newtons using the unit ratio: $\frac{1\text{ N}}{0.2248089\text{ lb}}$ $w = 7{,}275\ \cancel{\text{lb}} \cdot \frac{1\text{ N}}{0.2248089\ \cancel{\text{lb}}}$ $w = 32{,}360.8185\text{ N}$ $\mathbf{w \approx 32{,}360.8\text{ N}}$	$\mathbf{w = 7{,}275\text{ lb}}$

As mentioned on page 33, when working with weight, the force equation can be rewritten as:

$$w = m\ g$$

Since we know the value for weight (w) and the value for gravity (g), we can rewrite this equation to find the ***mass*** (m) of the spacecraft as follows:

$$m = \frac{w}{g}$$

When solving for mass, our units will need to be in kilograms (kg) for SI or slugs for Customary. Let's take another look at the relationship between mass and weight for both systems of units.

SI: Relationship between weight in newtons and mass in kilograms

$$F = m\ a$$

$$F \text{ (newton)} = m \text{ (kilogram)} \bullet a \text{ (meters/second/second)}$$

Special case (for acceleration due to gravity only):

$$w = m\ g$$

$$w \text{ (newton)} = m \text{ (kilogram)} \bullet g \text{ (meters/second/second)}$$

On Earth: $$w \text{ (newton)} = m \text{ (kilogram)} \bullet 9.8 \text{ meters/second/second}$$

To solve for mass: $$m = \frac{w}{g}$$

$$m \text{ (kilogram)} = \frac{w \text{ (newton)}}{9.8 \text{ meters/second/second)}}$$

The corresponding mass (m) in kilograms of a weight (w) in newtons is determined by dividing the weight (w) by gravity (g), which on Earth is 9.8 meters/second/second.

Customary: Relationship between weight in pounds and mass in slugs

$$F = m\ a$$

$$F\ (\text{pound}) = m\ (\text{slug}) \cdot a\ (\text{feet/second/second})$$

Special case (for acceleration due to gravity only):

$$w = m\ g$$

$$w\ (\text{pound}) = m\ (\text{slug}) \cdot g\ (\text{feet/second/second})$$

On Earth:

$$w\ (\text{pound}) = m\ (\text{slug}) \cdot 32.2\ \text{feet/second/second}$$

To solve for mass:

$$m = \frac{w}{g}$$

$$m\ (\text{slug}) = \frac{w\ (\text{pound})}{32.2\ \text{feet/second/second}}$$

The corresponding mass (m) in slugs of a weight (w) in pounds is determined by dividing the weight (w) by gravity (g), which on Earth is 32.2 feet/second/second.

Now we are ready to solve for the ***mass*** (m) of the spacecraft:

$$m = \frac{w}{g}$$

SI	Customary
$m = \dfrac{32{,}360.8\ \text{N}}{9.80665\ \text{m/s}^2}$	$m = \dfrac{7{,}275\ \text{lb}}{32.2\ \text{ft/s}^2}$
$m = \dfrac{32{,}360.8\ \text{kg}\ (\cancel{\text{m/s}^2})}{9.80665\ \cancel{\text{m/s}^2}}$	$m = \dfrac{225.93\ \text{lb}}{\text{ft/s}^2}$
$m = 3{,}299.88\ \text{kg}$	$m \approx \dfrac{226\ \text{lb}}{\text{ft/s}^2}$
	or
$m \approx 3{,}300\ \text{kg}$	**$m \approx 226\ \text{slugs}$**

We can solve for ***acceleration*** (resulting from firing the thrusters) by rewriting the equation again so that force is divided by mass.

$$a = \frac{F}{m}$$

SI	Customary
$a = \dfrac{44.5 \text{ N}}{3{,}300 \text{ kg}}$	$a = \dfrac{10 \text{ lb}}{226 \text{ slugs}}$
$a = \dfrac{44.5 \cancel{\text{kg}} \, (\text{m/s}^2)}{3{,}300 \cancel{\text{kg}}}$	$a = 10 \cancel{\text{lb}} \cdot \dfrac{\text{ft/s}^2}{226 \cancel{\text{lb}}}$
$a = 0.01348 \text{ m/s}^2$	$a = 0.04425 \text{ ft/s}^2$
$\mathbf{a \approx 0.0135 \text{ m/s}^2}$	$\mathbf{a \approx 0.044 \text{ ft/s}^2}$

For this situation, the spacecraft needs to change (in this case, *increase*) ***velocity*** such that it will travel 10 additional kilometers over an 8 hour period. We can write this change in velocity (delta-V) as:

$$\text{delta-V} = \frac{10 \text{ km}}{8 \text{ hr}}$$

To make future calculations more manageable, we can write delta-V as a decimal.

$$\text{delta-V} = \frac{10 \text{ km}}{8 \text{ hr}}$$

$$\text{delta-V} = 1.25 \text{ km/hr}$$

Next, we can convert the units of from kilometers per hour (km/hr) to meters per second (m/sec) for SI and to feet per second (ft/s) for Customary.

SI	Customary
First, convert kilometers to meters using the unit ratio: $\dfrac{1{,}000 \text{ meters}}{1 \text{ kilometer}}$	First, convert kilometers to feet using the unit ratio: $\dfrac{3{,}280.8399 \text{ feet}}{1 \text{ kilometer}}$
$\text{delta-V} = \dfrac{1.25 \cancel{\text{km}}}{1 \text{ hr}} \cdot \dfrac{1{,}000 \text{ m}}{1 \cancel{\text{km}}}$	$\text{delta-V} = \dfrac{1.25 \cancel{\text{km}}}{1 \text{ hr}} \cdot \dfrac{3{,}280.8399 \text{ ft}}{1 \cancel{\text{km}}}$
$= 1{,}250 \text{ m/hr}$	$= 4{,}101.04988 \text{ ft/hr}$

SI	Customary
Then, convert hours to seconds using the unit ratio:	Then, convert hours to seconds using the unit ratio:
$\frac{1 \text{ hour}}{3{,}600 \text{ seconds}}$	$\frac{1 \text{ hour}}{3{,}600 \text{ seconds}}$
delta-V $= \frac{1{,}250 \text{ m}}{1 \cancel{\text{hr}}} \bullet \frac{1 \cancel{\text{hr}}}{3{,}600 \text{ s}}$	delta-V $= \frac{4{,}101.04988 \text{ ft}}{1 \cancel{\text{hr}}} \bullet \frac{1 \cancel{\text{hr}}}{3{,}600 \text{ s}}$
delta-V = 0.3472 m/s	delta-V = 1.1392 ft/s
delta-V ≈ 0.35 m/s	**delta-V ≈ 1.14 ft/s**

In this problem situation, velocity can be thought of as acceleration times time. This is written as:

$$v = a\ t$$

Now that we know the change in velocity (v) and the rate of acceleration (a), we can solve for ***time*** (t) by rewriting the equation " v = a t " as:

$$t = \frac{v}{a}$$

SI	Customary
$t = \frac{0.35 \cancel{\text{m/s}}}{0.0135 \cancel{\text{m/s}}^2}$	$t = \frac{1.14 \cancel{\text{ft/s}}}{0.044 \cancel{\text{ft/s}}^2}$
t = 25.9259 s	t = 25.90909 s
t ≈ 25.9 seconds	**t ≈ 25.9 seconds**

Solution

The thrusters will need to be fired for 25.9 seconds in order to change the course of the spacecraft by 10 kilometers over an 8 hour period.

Note: Answers will vary based on approximation and rounding.

Credits and Sources

The information in the Problem Situation and Solution section (pp. 31–39) is attributed to the following sources:

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